Appendix D

Diameter Analysis

Buckling

(A MathCad Model)

by

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Diameter Analysis

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The purpose of this calculation is to determine optimum outer diameters for the probe casings, to prevent buckling direct push and sonic load. This in not a formal analysis and need not follow MCP-2374 for format, checking, etc. The results of this analysis support using a 2.5" outer diameter for the Lysimeter, Tensiometer, and Vapor Port (Conesipper). In the case of the Soil Moisture Probe, where constrained to either 1.75" or 2" off-the-shelf probe casings, the 2" is better.

References:

1. J. Shigley & C. Mischke, Mechanical Engineering Design, Fifth Edition, McGraw-Hill Book Company, 1989, ISBN 0-07-056899-5.
2. ASTM A276, "Standard Specification for Stainless and Heat-Resisting Steel Bars and Shapes," 1992.
3. ASTM A36, "Standard Specification for Carbon Structural Steel," 1997.
4. Ryerson, "Stock List," Data Section.

Approach [1, Sections 3-14 & 3-15]:

1. Compute $\left(\frac{L}{k}\right)_1$ & $\left(\frac{L}{k}\right)_{actual}$ $\quad \left(\frac{L}{k}\right)_1 = \sqrt{\frac{2 \cdot p^2 \cdot C_R \cdot E}{F_y}}$ [1, eqn 3-56]

2. If $\left(\frac{L}{k}\right)_1 \geq \left(\frac{L}{k}\right)_{actual}$ => Euler Eqn $\frac{P_{cr}}{A} = \frac{C_R \cdot p^2 \cdot E}{\left[\left(\frac{L}{k}\right)_{actual}\right]^2} = \frac{F_y}{2}$ [1, eqn 3-55]

$$P_{cr} = \frac{C_R \cdot p^2 \cdot E \cdot I}{L^2}$$

3. If $\left(\frac{L}{k}\right)_1 < \left(\frac{L}{k}\right)_{actual}$ => Johnson Eqn $\frac{P_{cr}}{A} = F_y - \left(\frac{F_y}{2 \cdot p}\right)^2 \cdot \frac{1}{C_R \cdot E} \cdot \left[\left(\frac{L}{k}\right)_{actual}\right]^2$ [1, eqn 3-58]

Materials: $\quad$ ORIGIN := 1 $\quad$ kip := 1000·lbf $\quad$ ksi := 1000·psi

$E_{sst} := 27.5 \cdot 10^6 \cdot psi$ $\quad$ Modulus of Elasticity for SST

$Fu_{sst} := 75 \cdot ksi$ $\quad$ $Fy_{sst} := 30 \cdot ksi$ $\quad$ Ultimate & yield strength for 304/316 SST per ASTM A276 [2], used in ARA's instrument casing.

$E_{cs} := 30 \cdot 10^6 \cdot psi$ $\quad$ Modulus of Elasticity for CS

$Fu_{413} := 86 \cdot ksi$ $\quad$ $Fy_{413} := 56 \cdot ksi$ $\quad$ Ultimate & yield strength for AISI 4130 [4], used in ARA's instrument casing, per Wes Bratton telecom (7-7-2000).

$Fu_{434} := 101 \cdot ksi$ $\quad$ $Fy_{434} := 69 \cdot ksi$ $\quad$ Ultimate & yield strength for AISI 4340 [4], used in ARA's probe casing, per Wes Bratton telecom (7-7-2000).

$$E := \begin{bmatrix} E_{sst} \\ E_{cs} \\ E_{cs} \end{bmatrix} \quad E = \begin{bmatrix} 2.75 \cdot 10^7 \\ 3 \cdot 10^7 \\ 3 \cdot 10^7 \end{bmatrix} \cdot psi \quad F_y := \begin{bmatrix} Fy_{sst} \\ Fy_{413} \\ Fy_{434} \end{bmatrix} \quad F_y = \begin{pmatrix} 30 \\ 56 \\ 69 \end{pmatrix} \cdot ksi$$

$C_R := 1.2$ Fixed-pinned condition

$Lki_{sst} := \sqrt{\frac{2 \cdot p^2 \cdot C_R \cdot E_{sst}}{Fy_{sst}}}$ $Lki_{sst} = 147.354$ Buckling Indice for Stainless Steel

$Lki_{413} := \sqrt{\frac{2 \cdot p^2 \cdot C_R \cdot E_{cs}}{Fy_{413}}}$ $Lki_{413} = 112.648$ Buckling Indice for AISI 4130 Carbon Steel (casing)

$Lki_{434} := \sqrt{\frac{2 \cdot p^2 \cdot C_R \cdot E_{cs}}{Fy_{434}}}$ $Lki_{434} = 101.483$ Buckling Indice for AISI 4340 Carbon Steel (instrument)

$$Lki := \begin{bmatrix} Lki_{sst} \\ Lki_{413} \\ Lki_{434} \end{bmatrix} \quad Lki = \begin{pmatrix} 147.35 \\ 112.65 \\ 101.48 \end{pmatrix}$$

SST/CS Tube:

1-3/4" OD x 3/8" wall Tube

$od_{175} := 1.75in$ $id_{175} := 1.0 \cdot in$ $w_{175} := 5.507 \cdot \frac{lbf}{ft}$ 1.75"x0.375" thk tube

$t_{175} := \frac{od_{175} - id_{175}}{2}$ $t_{175} = 0.375 \cdot in$

$A_{175} := \frac{p}{4} \cdot \left(od_{175}^2 - id_{175}^2\right)$ $A_{175} = 1.62 \cdot in^2$ $I_{175} := \frac{p \cdot \left(od_{175}^4 - id_{175}^4\right)}{64}$ $I_{175} = 0.41 \cdot in^4$

$k_{175} := \sqrt{\frac{I_{175}}{A_{175}}}$ $k_{175} = 0.504 \cdot in$

2" OD x 3/8" wall Tube

$od_{20} := 2.0 \cdot in$ $id_{20} := 1.25in$ $w_{20} := 6.508 \frac{lbf}{ft}$ 2.0"x0.325" thk tube

$t_{20} := \frac{od_{20} - id_{20}}{2}$ $t_{20} = 0.375 \cdot in$

$A_{20} := \frac{p}{4} \cdot \left(od_{20}^2 - id_{20}^2\right)$ $A_{20} = 1.91 \cdot in^2$ $I_{20} := \frac{p \cdot \left(od_{20}^4 - id_{20}^4\right)}{64}$ $I_{20} = 0.67 \cdot in^4$

$k_{20} := \sqrt{\frac{I_{20}}{A_{20}}}$ $k_{20} = 0.59 \cdot in$

$P_{mast} := 37300\,lbf$ Peak dynamic load along probe for 30,000-lbf & 120 Hz input

$L_s := 1 \cdot ft$ Unbraced pipe buckling length $r := 1..rows(E)$

$$Ls175_r := root\left(\left(\left(Lki_r \geq \frac{L_s}{k_{175}}, \frac{C_R \cdot p^2 \cdot E_r \cdot I_{175}}{L_s^2}, \left(F_{y_r} - \left(\frac{F_{y_r}}{2 \cdot p}\right)^2 \cdot \frac{\left(\frac{L_s}{k_{175}}\right)^2}{C_R \cdot E_r}\right) \cdot A_{175}\right) - 1.0 \cdot P_{mast} L_s\right)\right)$$

$$Ls20_r := root\left(\left(\left(Lki_r \geq \frac{L_s}{k_{20}}, \frac{C_R \cdot p^2 \cdot E_r \cdot I_{20}}{L_s^2}, \left(F_{y_r} - \left(\frac{F_{y_r}}{2 \cdot p}\right)^2 \cdot \frac{\left(\frac{L_s}{k_{20}}\right)^2}{C_R \cdot E_r}\right) \cdot A_{20}\right) - 1.0 \cdot P_{mast} L_s\right)\right)$$

$Ws175_r := w_{175} \cdot Ls175_r$ $Ws20_r := w_{20} \cdot Ls20_r$

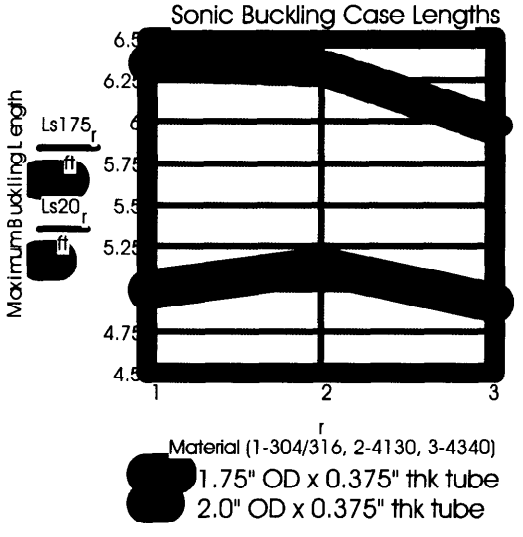
$Ls175^T = (4.99\ \ 5.13\ \ 4.92) \cdot ft$ $Ls20^T = (6.353\ \ 6.321\ \ 5.974) \cdot ft$ Sonic Peak Load Lengths

$Ls175^T = (59.93\ \ 61.6\ \ 59.03) \cdot in$ $Ls20^T = (76.233\ \ 75.851\ \ 71.685) \cdot in$

$s_{s175} := \frac{P_{mast}}{A_{175}}$ $s_{s175} = 23.026 \cdot ksi$ $s_{s20} := \frac{P_{mast}}{A_{20}}$ $s_{s20} = 19.484 \cdot ksi$

$\frac{s_{s175}}{Fy_{sst}} = 0.768$ $\frac{s_{s20}}{Fy_{sst}} = 0.649$

$Ws175 = \begin{pmatrix} 27.502 \\ 28.268 \\ 27.09 \end{pmatrix} \cdot lbf$ 304 SST 4130 CS 4340 CS $Ws20 = \begin{pmatrix} 41.34 \\ 41.14 \\ 38.88 \end{pmatrix} \cdot lbf$



$P_{push} := 11000\,lbf$ Direct push load magnitude

$L_p := 1 \cdot ft$ Unbraced pipe buckling length $r := 1..rows(E)$

$$Lp175_r := root\left(\left(\left(Lki_r \geq \frac{L_p}{k_{175}}, \frac{C_R \cdot p^2 \cdot E_r \cdot I_{175}}{L_p^2}, \left(F_{y_r} - \left(\frac{F_{y_r}}{2 \cdot p}\right)^2 \cdot \frac{\left(\frac{L_p}{k_{175}}\right)^2}{C_R \cdot E_r}\right) \cdot A_{175}\right) - 1.25 P_{push} \cdot L_p\right)\right)$$

$$Lp20_r := root\left(\left(\left(Lki_r \geq \frac{L_p}{k_{20}}, \frac{C_R \cdot p^2 \cdot E_r \cdot I_{20}}{L_p^2}, \left(F_{y_r} - \left(\frac{F_{y_r}}{2 \cdot p}\right)^2 \cdot \frac{\left(\frac{L_p}{k_{20}}\right)^2}{C_R \cdot E_r}\right) \cdot A_{20}\right) - 1.25 P_{push} \cdot L_p\right)\right)$$

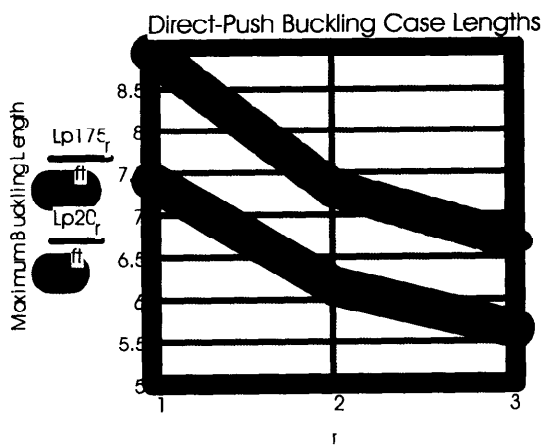
$Wp175_r := w_{175} \cdot Lp175_r$ $Wp20_r := w_{20} \cdot Lp20_r$

$Lp175^T = (7.41\ \ 6.16\ \ 5.64) \cdot ft$ $Lp20^T = (8.93\ \ 7.308\ \ 6.675) \cdot ft$ Direct-Push Load Lengths

$Lp175^T = (88.92\ \ 73.94\ \ 67.72) \cdot in$ $Lp20^T = (107.158\ \ 87.701\ \ 80.096) \cdot in$

$$Wp175 = \begin{pmatrix} 40.806 \\ 33.932 \\ 31.079 \end{pmatrix} \cdot lbf \qquad \begin{matrix} 304\ SST \\ 4130\ CS \\ 4340\ CS \end{matrix} \qquad Wp20 = \begin{pmatrix} 58.12 \\ 47.56 \\ 43.44 \end{pmatrix} \cdot lbf$$

Direct-Push Buckling Case Lengths



Material (1-304/316, 2-4130, 3-4340)

1.75" OD x 0.375" thk tube

2.0" OD x 0.375" thk tube

Summary:
Sonic Loading Safety Factor = 1.0 based on Fy (Material Yield Strength for minimum ASTM)
Direct-Push Loading Safety Factor = 1.25 based on Fy (Material Yield Strength - average)

Sonic Peak Load
Lengths SST 4130 4340

$Ls175^T = (4.99 \;\; 5.13 \;\; 4.92) \cdot ft$

$Ls175^T = (59.93 \;\; 61.6 \;\; 59.03) \cdot in$

$\frac{s_{s175}}{Fy_{sst}} = 0.768$ Maximum compression/tension stress OK

Direct-Push Load
Lengths SST 4130 4340

$Lp175^T = (7.41 \;\; 6.16 \;\; 5.64) \cdot ft$

$Lp175^T = (88.92 \;\; 73.94 \;\; 67.72) \cdot in$

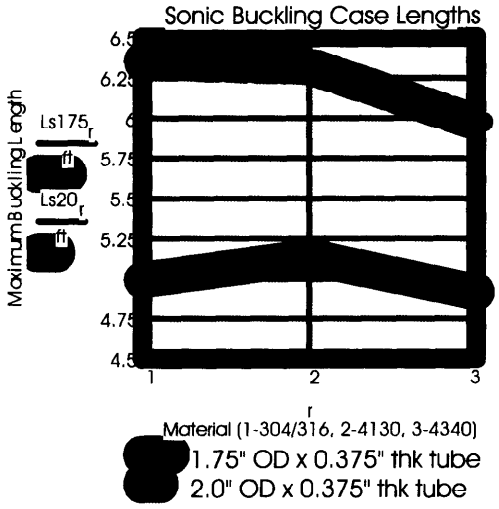
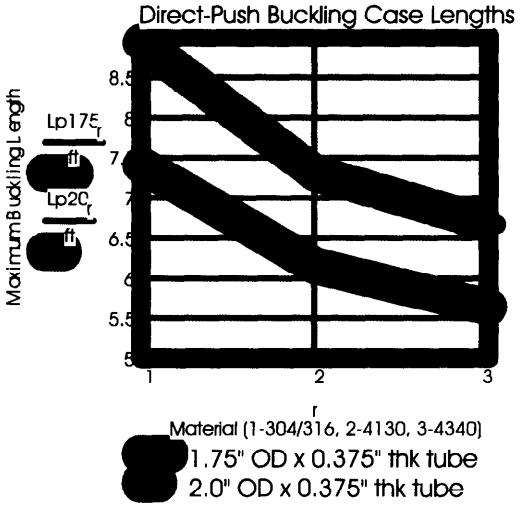
Rings Inst. Casing

$Ls20^T = (6.353 \;\; 6.321 \;\; 5.974) \cdot ft$

$Ls20^T = (76.233 \;\; 75.851 \;\; 71.685) \cdot in$

$\frac{s_{s20}}{Fy_{sst}} = 0.649$

Rings Inst. Casing

$Lp20^T = (8.93 \;\; 7.308 \;\; 6.675) \cdot ft$

$Lp20^T = (107.158 \;\; 87.701 \;\; 80.096) \cdot in$